Optimization of Power Grasps for Multiple Objects

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Abstract

Power grasp is a grasp that can hold objects stably without changing the joint torques of fingers. Almost all studies on power grasp deal with one object. But it is more efficient to hold multiple objects at the same time. This paper derives a condition for power grasp for multiple objects, and defines an optimal power grasp from the viewpoint of decreasing the work of joint torques. Lastly, we show some numerical examples to verify the validity of our approach.

1 Introduction

When a task is given of transferring a number of objects which are relatively small and light on some table to another table, we often grasp as many objects as we can simultaneously. This way is more efficient than to grasp the objects one by one and to transfer to the another table. This scheme can also be applied to operations by robot hands. Therefore, recently, there has been a growing interest in grasping of multiple objects simultaneously.

Aiyama et al. [1] have proposed a scheme for grasping multiple box type objects by two manipulators. Harada et al. [2] [3] have developed a method for grasping and manipulating multiple objects which make rolling contact with other objects or the links of fingers, and have studied equilibrium grasp and its robustness for multiple objects under gravitational field.

But many problems for grasping multiple objects still remain unclear. One of the problems is a research of power grasp. When some feasible joint torques have been assigned to finger joints in advance, the power grasp can automatically change its contact forces to resist magnitude-bounded external forces exerting on the object from any direction without changing the preloaded joint torques. Many researchers have studied about this power grasp for one object [4]∼[7].

Omata et al. [5] have given a kinematic condition for power grasp, and showed that contact sliding directions are constrained in power grasp. Zhang et al. [6] have provided a computational algorithm for calculating the critical external force which is required to move the grasped object in a definite direction. Yu [7] have given a necessary and sufficient condition for power grasp, have defined an optimal power grasp from the viewpoint of decreasing the magnitude of joint torques, and have developed the determination procedure of the optimal power grasp.

In this paper, we given a necessary and sufficient condition for forming power grasp and analyze its optimal power grasp when multiple objects are grasped by robot hands simultaneously. This paper is organized as follows. In sections 2, we give a condition for forming power grasp for multiple objects. Then, we define an optimal power grasp from the viewpoint of decreasing the magnitude of joint torques in section 3. Lastly, numerical examples are presented to show the effectiveness of our approach in section 4.

2 Condition for Power Grasp

In this section, we give a condition for forming power grasp for multiple objects. First, we formulate the kinematic constraint between a finger and an object and the one between an object and the other object. Then, we give the relationship between contact force applied to the object by the finger or the other object and joint torques or external force. Next, we show the frictional constraints, and finally we give a condition for forming power grasp. In the following discussion, we consider the problem in 3 dimensional space, but the obtained results can be applied to the problem in 2 dimensional space.

2.1 Target System

In this paper, we consider the cases where $M(\geq 1)$ objects are grasped by $N(\geq 1)$ fingers (**Fig.1**). We make the following assumption: (i) each object is a convex polyhedron; (ii) each link (or the fingertip) of the fingers makes frictional point-contact with the object's edge (or object's face); (iii) each object makes frictional surface(or line or point)-contact with the other object (or the base) and the surface(or line)-contact can approximately be represented by a number of point-contact; (iv) there exists at most one contact point on each link of the fingers. let Σ_R, Σ_{B_i}, and $\Sigma_{F_{jk}}$ be the reference coordinate frame, the object coordinate frame fixed at Object i, and the finger-link coordinate frame fixed at kth contact

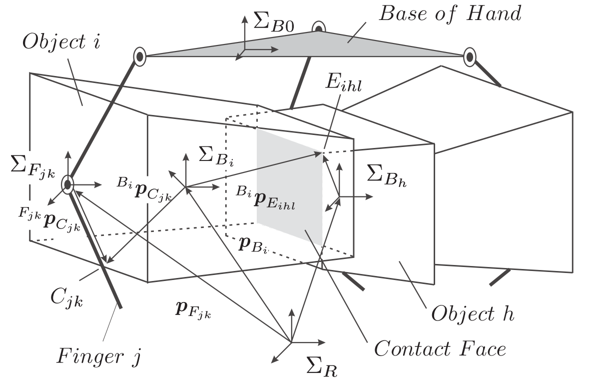

Figure 1: Robot hand and objects

point on the link of Finger j, respectively. Note that Object 0 means the base and Σ_{B_0} denotes the coordinate frame fixed at the base.

2.2 Kinematic Constraints

In this subsection, we formulate the kinematic constraint between the finger and the object andthe one between an object and the other object.

Let $\boldsymbol{p}_A$ and $\boldsymbol{R}_A$ be the position and orientation, respectively, of Σ_A with respect to Σ_R. let ${}^A\boldsymbol{p}_{C_{jk}}$ and ${}^A\boldsymbol{p}_{E_{ihl}}$ be the position of the kth contact point of Finger j C_{jk} and the position of the lth apex on the contact face between Object i and Object h, respectively, with respect to Σ_A.

Then, we get the following relationships [2] [3] [8].

$$\boldsymbol{D}_{B_{ijk}} \begin{bmatrix} \dot{\boldsymbol{p}}_{B_i} \\ \boldsymbol{\omega}_{B_i} \end{bmatrix} = \boldsymbol{D}_{F_{jk}} \begin{bmatrix} \dot{\boldsymbol{p}}_{F_{jk}} \\ \boldsymbol{\omega}_{F_{jk}} \end{bmatrix}, \quad (1)$$

$$\boldsymbol{D}_{E_{ihl}} \begin{bmatrix} \dot{\boldsymbol{p}}_{B_i} \\ \boldsymbol{\omega}_{B_i} \end{bmatrix} = \boldsymbol{D}_{E_{hil}} \begin{bmatrix} \dot{\boldsymbol{p}}_{B_h} \\ \boldsymbol{\omega}_{B_h} \end{bmatrix}, \quad (2)$$

where

$$\boldsymbol{D}_{B_{ijk}} = \left[\boldsymbol{I}_3 \; -\left[(\boldsymbol{R}_{B_i}{}^{B_i}\boldsymbol{p}_{C_{jk}})\times\right]\right] \in \boldsymbol{R}^{3\times 6},$$

$$\boldsymbol{D}_{F_{jk}} = \left[\boldsymbol{I}_3 \; -\left[(\boldsymbol{R}_{F_{jk}}{}^{F_{jk}}\boldsymbol{p}_{C_{jk}})\times\right]\right] \in \boldsymbol{R}^{3\times 6},$$

$$\boldsymbol{D}_{E_{ihl}} = \left[\boldsymbol{I}_3 \; -\left[(\boldsymbol{R}_{B_i}{}^{B_i}\boldsymbol{p}_{E_{ihl}})\times\right]\right] \in \boldsymbol{R}^{3\times 6}.$$

Here, $[a\times]$ denotes a skew symmetric matrix equivalent to the cross product operation, $\boldsymbol{I}_k$ denotes the k-order identity matrix and $\boldsymbol{\omega}_{B_i}$ and $\boldsymbol{\omega}_{F_{jk}}$ denote the angular velocities of Σ_{B_i} and $\Sigma_{F_{jk}}$, respectively. Note that (1) expresses the relationship between the velocity of $\Sigma_{F_{jk}}$ and the velocity of Σ_{B_i} with respect to the contact point C_{jk} and that (2) expresses the relationship between the velocity of Σ_{B_i} and the velocity of Σ_{B_h} with respect to the lth apex on the contact face between Object i and Object h.

Now, let $\boldsymbol{q}_j \in \boldsymbol{R}^{L_j}$ be a joint vector of Finger j where L_j denotes the number of joints of Finger j (Note that a joint will not be numbered if there is no contact point from the joint to the fingertip). Then, we get the following relationship between the joint velocity of Finger j and the velocity of $\Sigma_{F_{jk}}$

$$\begin{bmatrix} \dot{\boldsymbol{p}}_{F_{jk}} \\ \boldsymbol{\omega}_{F_{jk}} \end{bmatrix} = \boldsymbol{J}_{F_{jk}} \dot{\boldsymbol{q}}_j, \quad (3)$$

where $\boldsymbol{J}_{F_{jk}} \in \boldsymbol{R}^{6\times L_j}$ denotes a Jacobian matrix of Finger j.

Form (1)(3), we get the following relationship between the joint velocity of Finger j and the velocity of multiple objects

$$\boldsymbol{D}_{B_j} \left[\, \dot{\boldsymbol{p}}_{B_1}^T \;\; \boldsymbol{\omega}_{B_1}^T \;\; \cdots \;\; \dot{\boldsymbol{p}}_{B_M}^T \;\; \boldsymbol{\omega}_{B_M}^T \,\right]^T = \boldsymbol{J}_{CF_j} \dot{\boldsymbol{q}}_j, \quad (4)$$

where

$$\boldsymbol{J}_{CF_j} = \begin{bmatrix} \boldsymbol{D}_{F_{j1}} \boldsymbol{J}_{F_{j1}} \\ \vdots \\ \boldsymbol{D}_{F_{jK_j}} \boldsymbol{J}_{F_{jK_j}} \end{bmatrix} \in \boldsymbol{R}^{3K_j \times L_j},$$

$$\boldsymbol{D}_{B_j} = \begin{array}{cc} & \begin{array}{ccc} & (i) & \end{array} \\ \text{(k)} & \begin{bmatrix} \cdots & \vdots & \cdots \\ \boldsymbol{0} & \boldsymbol{D}_{B_{ijk}} & \boldsymbol{0} \\ \cdots & \vdots & \cdots \end{bmatrix} \end{array} \in \boldsymbol{R}^{3K_j \times 6M} .$$

Here, $\boldsymbol{D}_{B_j}$ denotes the matrix whose (k,i)th component is $\boldsymbol{D}_{B_{ijk}}$ and whose the other components are all $\boldsymbol{0}$ when we take only kth row of the matrix into account. Note that K_j denotes the number of contact points on Finger j.

Next, when Object i contacts with Object h and $i < h$, we get the following equation form (2).

$$\boldsymbol{D}_{E_i} \left[\, \dot{\boldsymbol{p}}_{B_1}^T \;\; \boldsymbol{\omega}_{B_1}^T \;\; \cdots \;\; \dot{\boldsymbol{p}}_{B_M}^T \;\; \boldsymbol{\omega}_{B_M}^T \,\right]^T = \boldsymbol{0}, \quad (5)$$

where

$$\boldsymbol{D}_{E_i} = \begin{array}{cc} & \begin{array}{ccccc} & (i) & & (h) & \end{array} \\ (l) & \begin{bmatrix} \cdots & \vdots & \cdots & \vdots & \cdots \\ \boldsymbol{0} & \boldsymbol{D}_{E_{ihl}} & \boldsymbol{0} & -\boldsymbol{D}_{E_{hil}} & \boldsymbol{0} \\ \cdots & \vdots & \cdots & \vdots & \cdots \end{bmatrix} \end{array} \in \boldsymbol{R}^{3T_{ih} \times 6M} .$$

Here, this matrix denotes the matrix whose (l,i)th component is $\boldsymbol{D}_{E_{ihl}}$, whose (l,h)th component is $-\boldsymbol{D}_{E_{hil}}$, and whose the other components are all $\boldsymbol{0}$, when we take only lth row of the matrix into account. Note that if $\boldsymbol{D}_{E_{ihl}} = \boldsymbol{0}$, it means that Object i is the base. Note also that T_{ih} denotes the number of apices of the contact face between Object i and Object h.

From(4),(5), we get the following relationship between the joint velocities of fingers and the velocities of multiple objects

$$\left[\, \boldsymbol{D}_B \;\; \boldsymbol{J}_{CF} \,\right] \begin{bmatrix} \dot{\boldsymbol{x}} \\ -\dot{\boldsymbol{q}} \end{bmatrix} = \boldsymbol{0}, \quad (6)$$

where

$$\boldsymbol{D}_B = \left[\, \boldsymbol{D}_{B_1}^T \;\; \cdots \;\; \boldsymbol{D}_{B_N}^T \;\; (\boldsymbol{D}_{E_0})^T \;\; \boldsymbol{D}_{E_1}^T \right.$$

$$\cdots\ \boldsymbol{D}_{E_{M-1}}^T \Big]^T \in \boldsymbol{R}^{3(K+T)\times 6M},$$

$$\boldsymbol{J}_{CF} = \begin{bmatrix} \boldsymbol{J}_{CF_1} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{J}_{CF_N} \\ (\boldsymbol{0} & \cdots & \boldsymbol{0}) \\ \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix} \in \boldsymbol{R}^{3(K+T)\times L},$$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{p}}_{B_1}^T & \boldsymbol{\omega}_{B_1}^T & \cdots & \dot{\boldsymbol{p}}_{B_M}^T & \boldsymbol{\omega}_{B_M}^T \end{bmatrix}^T \in \boldsymbol{R}^{6M},$$

$$\dot{\boldsymbol{q}} = \begin{bmatrix} \dot{\boldsymbol{q}}_1^T & \cdots & \dot{\boldsymbol{q}}_N^T \end{bmatrix}^T \in \boldsymbol{R}^{L}.$$

Note that the component in brackets ( ) in matrices will be taken off if there is no contact point or face on the base. Note also that $K = \Sigma_{i=1}^N K_i$, $T = \Sigma_{i,i<h} T_{ih}$, and $L = \Sigma_{i=1}^N L_i$.

2.3 Contact Force

In this subsection, we consider the relationship between contact force applied to the object by the finger or the other object and joint torques or external force applied to each object. Note that we assume that contact force between objects can be represented by the resultant force of all contact forces at the all apex on the contact face.

Let $\boldsymbol{f}_{C_{jk}} \in \boldsymbol{R}^3$ and $\boldsymbol{f}_{E_{ihl}} \in \boldsymbol{R}^3$ be a contact force at the contact point C_{jk} and the one at the apex on the contact faces between Object i and Object h E_{ihl}, respectively. Note that $\boldsymbol{f}_{E_{ihl}}$ denotes the force applied to Object i by Object h when Object h contacts with Object i and $i < h$. Now, let $\boldsymbol{\tau}_j \in \boldsymbol{R}^{L_j}$ and $\boldsymbol{t}_i \in \boldsymbol{R}^6$ be the joint torque vector of Finger j and the external force which is composed of 3 dimensional force and of 3 dimensional moment and applied to the origin of Σ_{Bi}, respectively. Then, from (6) and the principle of virtual work, we get

$$\boldsymbol{A}\boldsymbol{f} = \begin{bmatrix} -\boldsymbol{t}_e \\ \boldsymbol{\tau} \end{bmatrix}, \tag{7}$$

where

$$\boldsymbol{A} = [\boldsymbol{D}_B^T \ \ \boldsymbol{J}_{CF}^T]^T \in \boldsymbol{R}^{(6M+L)\times 3(K+T)},$$

$$\boldsymbol{f} = \Big[\boldsymbol{f}_{C_{11}}^T \cdots \boldsymbol{f}_{C_{NK_N}}^T (\cdots \boldsymbol{f}_{E_{021}}^T \cdots \boldsymbol{f}_{E_{0hT_{0h}}}^T \cdots),$$

$$\boldsymbol{f}_{E_{121}}^T \cdots \boldsymbol{f}_{E_{M-1MT_{M-1M}}}^T \Big]^T \in \boldsymbol{R}^{3(K+T)},$$

$$\boldsymbol{t}_e = \big[\boldsymbol{t}_1^T \cdots \boldsymbol{t}_M^T\big]^T \in \boldsymbol{R}^{6M},$$

$$\boldsymbol{\tau} = \big[\boldsymbol{\tau}_1^T \cdots \boldsymbol{\tau}_N^T\big]^T \in \boldsymbol{R}^{L}.$$

From (7), we get

$$\boldsymbol{f} = (\boldsymbol{J}_{CF}^+)^T \boldsymbol{\tau} + (\boldsymbol{I}_{3(K+T)} - \boldsymbol{J}_{CF}\boldsymbol{J}_{CF}^+)\boldsymbol{k}_1, \tag{8}$$

where $\boldsymbol{J}_{CF}^+ \in \boldsymbol{R}^{L\times 3(K+T)}$ denotes the pseudo-inverse matrix of $\boldsymbol{J}_{CF}$ and $\boldsymbol{k}_1$ denotes an arbitrary vector. If we assign the constant value $\boldsymbol{\tau}_C$ to $\boldsymbol{\tau}$ in advance, (8) is rewritten as follows

$$\boldsymbol{f} = (\boldsymbol{J}_{CF}^+)^T \boldsymbol{\tau}_C + (\boldsymbol{I}_{3(K+T)} - \boldsymbol{J}_{CF}\boldsymbol{J}_{CF}^+)\boldsymbol{k}_1. \tag{9}$$

$\boldsymbol{f}$ given by (9) is a contact force which can occur without changing the value of pre-loaded joint torques $\boldsymbol{\tau}_C$. Note that the force of the second term in the right side of (9) expresses the set of the internal force which exerts no influence on the joint torques. Then, the set of contact force $\boldsymbol{f}$ satisfying (9) is given by

$$\mathcal{F}_J = \{\boldsymbol{f} | \boldsymbol{f} = (\boldsymbol{J}_{CF}^+)^T \hat{\boldsymbol{\tau}}_C k_c + (\boldsymbol{I}_{3(K+T)} - \boldsymbol{J}_{CF}\boldsymbol{J}_{CF}^+)\boldsymbol{k}_1,$$
$$k_c > 0,\ \boldsymbol{k}_1 \in \boldsymbol{R}^{3(K+T)}\}, \tag{10}$$

where $\hat{\boldsymbol{\tau}}_C (= \boldsymbol{\tau}_C / \|\boldsymbol{\tau}_C\|)$ and $k_C (= \|\boldsymbol{\tau}_C\|)$ denotes the direction and the magnitude of $\boldsymbol{\tau}_C$, respectively.

2.4 Frictional Constraints

In this subsection, we consider the frictional constraint at the contact point C_{jk} and the one at the apex on the contact face E_{ihl}. Here, we assume that that the frictional constraint at the contact face can be satisfied if all frictional constraints at all apices on the contact face are all satisfied.

Then, contact force $\boldsymbol{f}_{C_{jk}}$ and $\boldsymbol{f}_{E_{ihl}}$ must satisfy the following frictional constraints at C_{jk} and E_{ihl}, respectively.

$$\boldsymbol{n}_F^T \boldsymbol{f}_F \geq \frac{1}{\sqrt{1+\mu_F^2}} \|\boldsymbol{f}_F\|, \tag{11}$$

where F means C_{jk} or E_{ihl}, and $\mu_{C_{jk}}$ and $\mu_{E_{ihl}}$ denote the coefficient of maximum static friction at C_{jk} and E_{ihl}, respectively. Hence, the set of $\boldsymbol{f}_{C_{jk}}$ and $\boldsymbol{f}_{E_{ihl}}$ satisfying (11) at all contact points and apices is given by

$$\mathcal{F}_f = \{\ \boldsymbol{f} \mid \boldsymbol{N}^T \boldsymbol{f} \geq \hat{\boldsymbol{\mu}} \bar{\boldsymbol{f}}\ \}, \tag{12}$$

where

$$\boldsymbol{N} = \mathrm{diag}\Big[\boldsymbol{n}_{C_{11}} \cdots \boldsymbol{n}_{C_{NK_N}} \ (\cdots \boldsymbol{n}_{E_{01}} \cdots \boldsymbol{n}_{E_{0hT_{0h}}} \cdots)$$
$$\boldsymbol{n}_{E_{121}} \cdots \boldsymbol{n}_{E_{M-1MT_{M-1M}}}\Big] \in \boldsymbol{R}^{3(K+T)\times 3(K+T)},$$

$$\hat{\boldsymbol{\mu}} = \mathrm{diag}\Big[(1+\mu_{C_{11}}^2)^{-\frac{1}{2}} \cdots (1+\mu_{C_{NK_N}}^2)^{-\frac{1}{2}}$$
$$(\cdots (1+\mu_{E_{01}}^2)^{-\frac{1}{2}} \cdots (1+\mu_{E_{0MT_{0M}}}^2)^{-\frac{1}{2}} \cdots)$$
$$(1+\mu_{E_{121}}^2)^{-\frac{1}{2}} \cdots (1+\mu_{E_{M-1MT_{M-1M}}}^2)^{-\frac{1}{2}}\Big]$$
$$\in \boldsymbol{R}^{(K+T)\times(K+T)},$$

$$\bar{\boldsymbol{f}} = \Big[\|\boldsymbol{f}_{C_{11}}\| \ \ldots\ \|\boldsymbol{f}_{C_{NK_N}}\|$$
$$(\ldots \|\boldsymbol{f}_{E_{01}}\| \ \ldots\ \|\boldsymbol{f}_{E_{0MT_{0M}}}\| \ldots)$$
$$\|\boldsymbol{f}_{E_{121}}\| \ \ldots\ \|\boldsymbol{f}_{E_{M-1MT_{M-1M}}}\| \Big]^T$$
$$\in \boldsymbol{R}^{(K+T)}.$$

Here, "diag" means a block diagonal matrix.

2.5 Condition for Power Grasp

In this subsection, we derive a necessary and sufficient condition for forming power grasp for multiple objects.

The contact force, which can actually occur, is the force that not only satisfies the frictional constraints at the contact point or the apex on the contact face but also is contained in the set expressed by (12). Then, from (10) and (12), the set of the above contact forces is given by

$$\mathcal{F} \quad = \quad \mathcal{F}_J \ \cap \ \mathcal{F}_f . \tag{13}$$

When we consider whether the system can form power grasp or not, the direction of possible contact force to occur is the problem. So, if we set $\hat{\boldsymbol{\tau}}_C$ is constant and k_c and $\boldsymbol{k}_1$ can change in (10), $\mathcal{F}_J$ become a convex corn. Hence, Since $\mathcal{F}_f$ is also a convex corn from [7], we can regard $\mathcal{F}$ as a convex corn. $\boldsymbol{t}_e$, given by the substitution of $\boldsymbol{f}$ satisfying (13) into (7), is an external force which can be resisted without changing the direction of pre-loaded joint torque $\boldsymbol{\tau}_C$. Then, the set composed of this $\boldsymbol{t}_e$ can be expressed by

$$\mathcal{W} \quad = \quad \{ \ \boldsymbol{t}_e \mid \boldsymbol{t}_e \ = \ -\boldsymbol{D}_B^T \boldsymbol{f} \ , \ \boldsymbol{f} \in \mathcal{F} \ \}. \tag{14}$$

Note that $\mathcal{W}$ is also a convex corn, since $\mathcal{F} \rightarrow \mathcal{W}$ is a linear mapping. Hence the linear space of $\mathcal{W}$ is given by

$$\widehat{\mathcal{W}} = \mathcal{W} \ \cap \ (-\mathcal{W}). \tag{15}$$

This linear space $\widehat{\mathcal{W}}$ expresses the set of resistible external forces exerted from the bilateral direction.

On the other hand, the rank of $\boldsymbol{A}$ in (7) is also important to form power grasp. If rank$\boldsymbol{A} < (6M + L)$, contact force $\boldsymbol{f}$ can not be determined even when some constant value is given to $\boldsymbol{\tau}$ and some external force $\boldsymbol{t}_e$ apply to the system. This means a contact force for compensating some external force cannot occur and that then, the system cannot form power grasp.

From the definition of power grasp, when M objects are grasped simultaneously by robot hands and both rank$\boldsymbol{A}$=$6M + L$ and dim$\widehat{\mathcal{W}}$=$6M$ (dim$\widehat{\mathcal{W}}$ denotes the number of the dimension of $\widehat{\mathcal{W}}$) are satisfied, the grasp can become power grasp. Hence, when M objects are grasped simultaneously by robot hands and the directions of some feasible joint torques are assigned to finger joints in advance, a necessary and sufficient condition for the existence of power grasp is given by

1. $\widehat{\mathcal{W}}$=$6M$

2. rank$\boldsymbol{A}$=$6M + L$.

When the system is in 2 dimensional plan, the above condition is rewritten by

1. $\widehat{\mathcal{W}}$=$3M$

2. rank$\boldsymbol{A}$=$3M + L$.

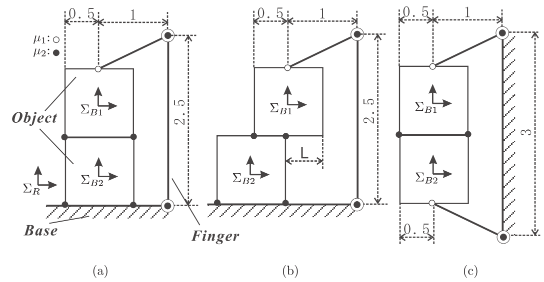

Figure 2: Examples

because the external force is composed of 2 dimensional force and of 1 dimensional moment.

Fig.2 shows some examples in planar motion. In this figure, each object is a square whose side is 1 length and we let the all coefficients of maximum static friction at all contact point between the finger and the object μ_1 and all ones at all apices on the contact face between the objects μ_2 be 0.3. The system shown in Fig.2 (a) can form power grasp. The system shown in Fig.2 (b) can form power grasp when $0 \leq L < 0.8$, but can not form when $L \geq 0.8$. The system shown in Fig.2 (c) cannot form power grasp because dim$\hat{\boldsymbol{W}}$=5 (< 6).

3 Optimal Power Grasp

In this section, we define an optimal power grasp for multiple objects in the same way as the definition of the optimal power grasp for one object given by Y. Yu [7].

When we simultaneously grasp multiple objects with power grasp by robot hands, there are an infinite number of power grasp forms. Thereby, it is necessary to select the most suitable one among the many power grasp forms by some evaluations. So, we use *Required External Force Set* in [7]. First, we define *Critical External Force Set* for the definition of *Required External Force Set* as follows.

Critical External Force Set *when the system form power grasp and some external force apply to the objects, the balancing contact force, which counteract the external force, can occur without changing the value of pre-loaded joint torques by the mechanism itself. However, the magnitude of the resistible external force is upper-bounded. We call the upper-bounded force Critical External Force and the set composed of the all upper-bounded forces Critical External Force Set* $\mathcal{T}_L \subset \boldsymbol{R}^{6M}$.

With this definition, we define *Required External Force Set* as follows.

Required External Force Set *Required External Force Set,* $\mathcal{T}_R \subset \boldsymbol{R}^{6M}$*, is a set which Critical External Force Set of the power grasp must contain.*

We think it is suitable that an assigned joint

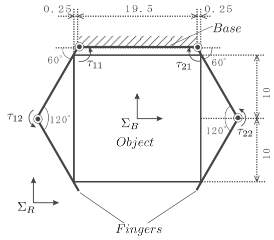

Figure 3: Robot hand and one object

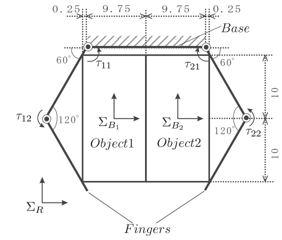

Figure 5: Robot hand and two objects(II)

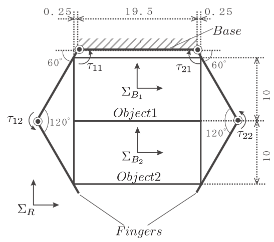

Figure 4: Robot hand and two objects(I)

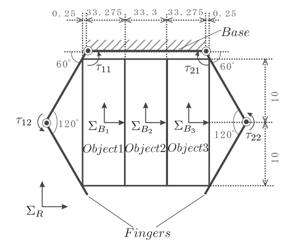

Figure 6: Robot hand and three objects

torque in advance is as small as possible. So, we give the following definition of an optimal power grasp.

Optimal power grasp *When pre-loaded torque of kth joint of Finger j is given by τ_{jk} ($|\tau_{jk}| \leq \tau_{jkmax}$) and τ_{jk} in the system, which forms a power grasp whose Critical External Force Set contains Required External Force Set ($\mathcal{T}_R \subset \mathcal{T}_L$), minimizes the following criterion function, we call the power grasp optimal power grasp.*

$$\Phi_1 \quad \triangleq \quad \max_{j\ ,\ k} \frac{|\tau_{jk}|}{\tau_{jkmax}} \tag{16}$$

Note that if all τ_{jkmax} are same, the above function can be expressed by

$$\Phi_2 \quad \triangleq \quad \max_{j\ ,\ k} |\tau_{jk}|. \tag{17}$$

The procedure for determining optimal power grasp in several numerical examples in the next section is the same as one proposed by Y. Yu [7]. So, we introduce the outline here.

From(7), we get

$$\boldsymbol{f} = (\boldsymbol{A}^{+})^{T} \begin{bmatrix} -\boldsymbol{t}_e \\ \boldsymbol{\tau} \end{bmatrix} + (\boldsymbol{I}_{3(K+T)} \ - \ \boldsymbol{A}\boldsymbol{A}^{T})\boldsymbol{k}_2, \tag{18}$$

where $\boldsymbol{A}^{+}$ denotes the pseudo-inverse matrix of $\boldsymbol{A}$ and $\boldsymbol{k}_2$ denotes an arbitrary vector. By evaluating whether the $\boldsymbol{A}$ and $\boldsymbol{\tau}$ satisfy both (12) and (18) or not with respect to all values of $\boldsymbol{A}$ and $\boldsymbol{\tau}$, we search $\boldsymbol{A}$ and $\boldsymbol{\tau}$ minimizing Φ_1 (Φ_2).

4 Numerical Examples

Based on the above discussion, in this section, we consider evaluating optimal power grasp of examples

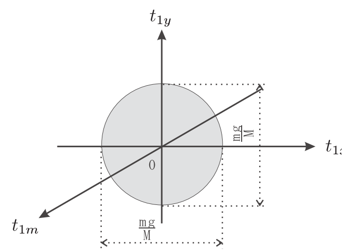

Figure 7: *Required external force set* for Fig.3

in planar motion shown in **Fig.3 ∼ Fig.6**. For the convenience, we make the following assumption; 1) Each object shown in Fig.3 ∼ FIg.6 has an uniform and same density. 2) Σ_{Bi} (the object coordinate frame fixed at the Object i) is fixed at the center of gravity of Object i. 3) the coefficients of maximum static friction at the contact point between the finger and the object $\mu_{C_{jk}}$ and the ones at the apex on the contact faces between the objects $\mu_{E_{ihl}}$ are all set to 0.3. 4) We make $\tau_{11} = -\tau_{21},\ \tau_{12} = -\tau_{22}$ from the bilateral symmetry of the configuration of multiple objects and robot hands. 5) The magnitudes of the maximum torques which can be actuated by the joints are all same and then, we can use the criterion function Φ_2 in (17).

letting t_{ix} and t_{iy} be the components of the external force applied to Object i and t_{im} be the external moment applied to Object i, each *Required external force set* for each system is given by

$$\mathcal{T}_R \quad = \quad \{t_{ix} = t_x,\ t_{iy} = t_y,\ t_{im} = 0 | \sqrt{t_x^2 + t_y^2} \leq \frac{mg}{M}\}, \tag{19}$$

where, m denotes the summation of all weights of M objects and g denotes the acceleration of gravity. For example, *Required external force set* for the

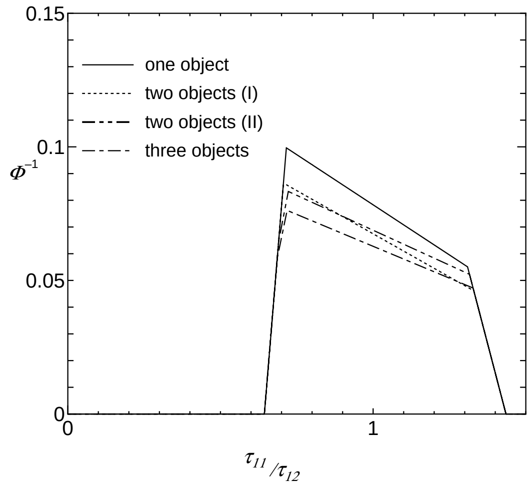

Figure 8: Φ_{2min}^{-1}

Table 1: Φ_{2min} and joint torques for optimal power grasp

	Φ_{2min}	$\|\tau_{11}\| = \|\tau_{21}\|$	$\|\tau_{12}\| = \|\tau_{22}\|$
one object	10.04	7.184	10.04
two objects(I)	11.66	8.234	11.66
two objects(II)	12.00	8.669	12.00
three objects	13.12	9.426	13.12

system shown in Fig.3 is shown in **Fig.7**. By using this *Required external force set*, we can achieve the requirement that we can retain power grasp even if the robot hand is in any configuration, when each system is laid in the same gravitational field.

Note that we set to $mg = 1.5$ and we approximate the circle shown in Fig.7 as the regular polygon whose sides are 64, in our actual computation.

Results are shown in **Fig.8** and **Table.1**. Fig.8 shows the reciprocal of Φ_{2min} which is the least value of Φ_2, when the value of τ_{11}/τ_{12} changes. The larger the value of Φ_{2min}^{-1} is, the smaller joint torques are necessary to form power grasp. $\Phi_{min}^{-1} = 0$ means the system cannot form power grasp. Table.1 shows the values of joint torques and Φ_{2min} where each system forms optimal power grasp, namely, where the value of Φ_{2min} become the least one.

From Fig.8 and Table.1, we can see that the necessary joint torques depends much on the number of objects in order to form power grasp and that the necessary joint torques where two objects are combined in up and down are smaller than where two objects are combined in left and right in order to form optimal power grasp. However, we can also see that the necessary torques where two objects are combined in up and down are smaller than where two objects are combined in left and right when $\tau_{11}/\tau_{12} < 0.8924$ and that the necessary torques where two objects are combined in left and right are smaller than where two objects are combined in up and down when $\tau_{11}/\tau_{12} \geq 0.8924$, in order to form power grasp.

5 Conclusions

In this paper, we have derived a necessary and sufficient condition for forming power grasp where multiple objects are grasped simultaneously by robot hands. We have also defines an optimal power grasp in terms that the grasp whose necessary magnitudes of joint torques to form power grasp is the smallest is optimal. Finally, we show some numerical examples in order to verify effectiveness of our approach.

From the result of the numerical examples, we can see that the necessary magnitudes of joint torques to form power grasp depends much on the number of objects grasped simultaneously.

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